Entre Piqures et Captures : Modelisation et Inférence de la Dynamique d'*Aedes* avec Controle par Insecte Sterile

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The Challenge of *Aedes* Population Control

Two invasive species dominate global expansion:

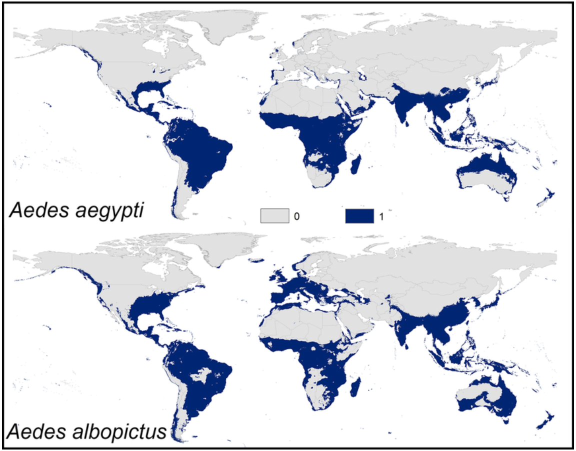


Figure: 2018 repartition of the two main *Aedes* Mosquitoes, from Mapping the global potential distributions of two arboviral vectors Aedes aegypti and Ae. albopictus under changing climate

France: Aedes Already Endemic

Main invasive *Aedes* species:

- **Réunion Island:** *Aedes albopictus* and *aegypti*.
- **French Polynesia:** *Aedes polynesiensis* and *aegypti*.
- **French Antilles (Martinique, Guadeloupe):** *Aedes aegypti*.
- **Mayotte:** *Aedes albopictus*.
- **Mainland France:** *Aedes albopictus*.

Mosquitoes and Disease Transmission

Figure: Close-up on the Massy mosquito.

Conventional Mosquito Control Strategies

Common approaches to limit mosquito-borne diseases, acting on:

1. **The disease → vaccination**
2. **The vector → insecticides**
3. **The environment → breeding site removal**

Vector-Specific Intervention: Sterile Insect Technique

Sterile Insect Technique (SIT):

- Release of sterilized males into the environment.
- These males compete with wild males for mating.
- Females mating with sterile males produce non-viable eggs.

Figure: SIT intervention.

Tetiaroa Atoll

Tetiaroa atoll in French Polynesia, provided by the Institut Louis Malardé (ILM).

Population monitoring performed on each motus. Focus on:

- Onetahi and Honuea motus
- Presence of *Aedes polynesiensis*

Figure: Map of the Tetiaroa atoll.

IIT intervention on the Tetiaroa Atoll

We have the weekly number of released incompatible males. During the monitoring, we do not have access to the male status.

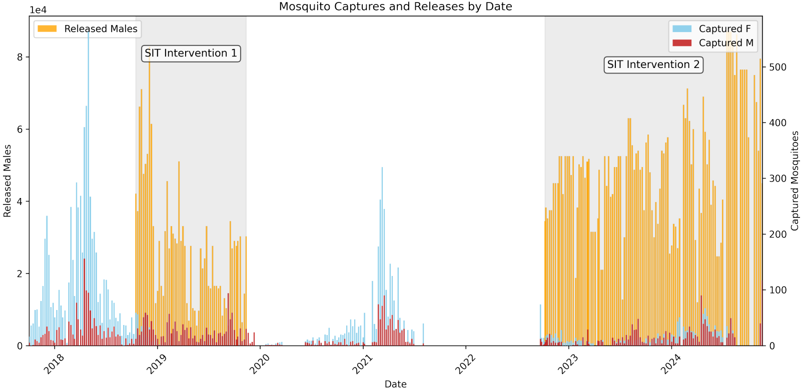


Figure: Captured and released mosquitoes on Onetahi Island.

Environmental data

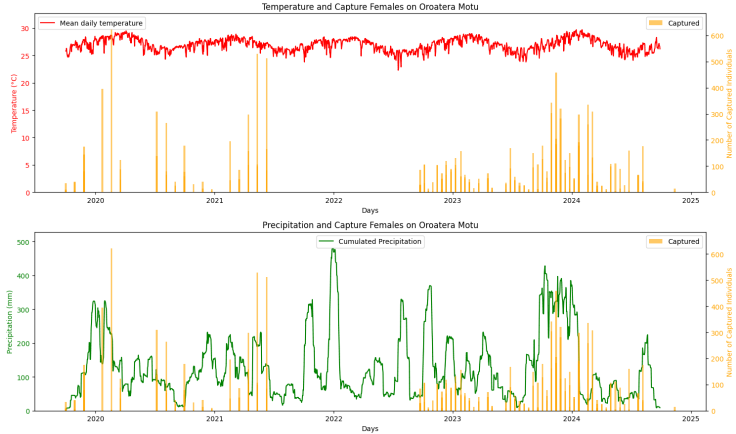
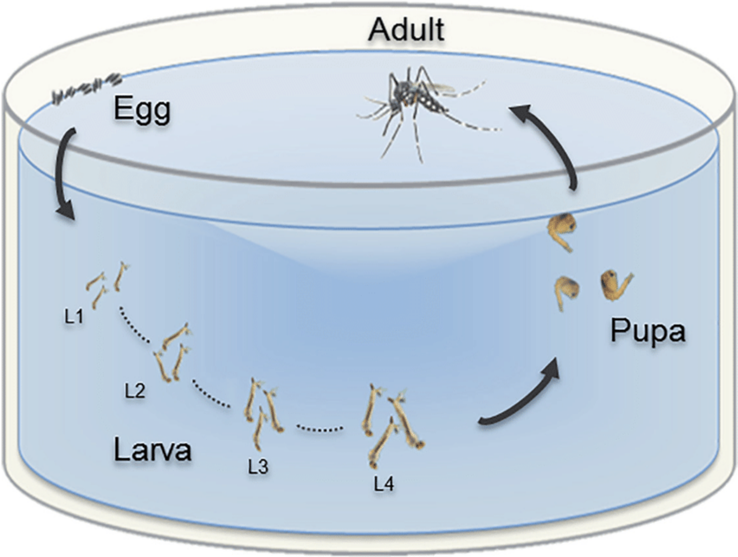


Figure: Environmental variables (temperature and monthly cumulative rainfall) compared with the number of captured females each week.

Mosquito Life Cycle



First model

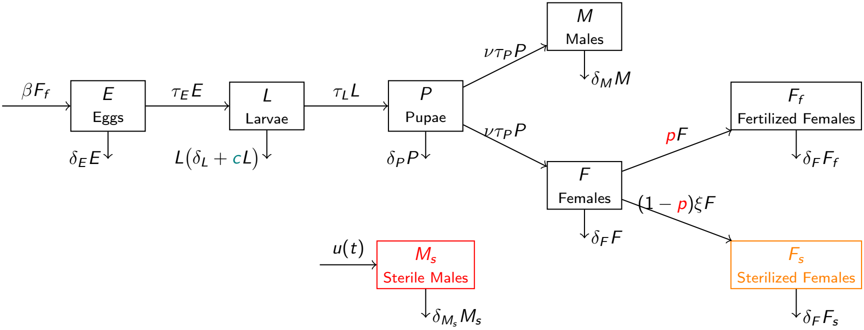


Figure: Mosquito life cycle compartmental model with IIT intervention

Population Process

Population vector: $(\mathrm{Z}(t))_{t\geq 0}$. This defines a time inhomogeneous Markov jump process with finite jumps.

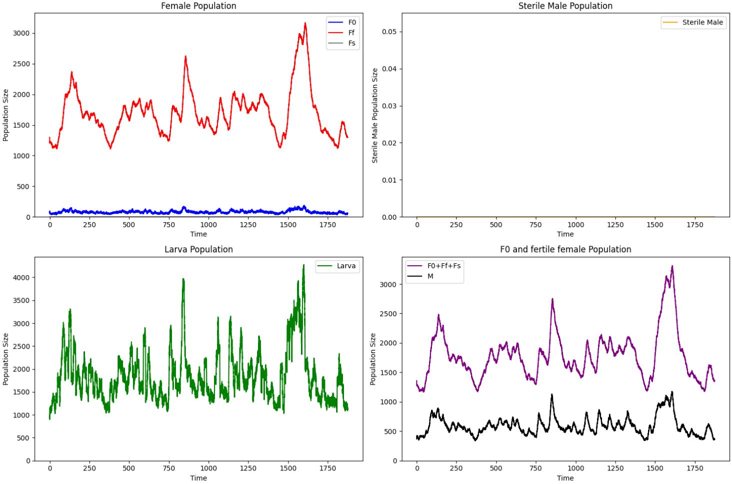
Let $\mathrm{J} \in \mathbb{Z}^8$ a finite index set. Each biological event $j \in \mathrm{J}$ is associated with a jump vector $\nu_j \in \mathbb{N}^d$ and an intensity $a_j(t, \mathrm{Z}(t))$.
Four types of events can happen to the population.

- **Birth in the egg state:** This is the only event where individuals are added to the system without human intervention.
- **Death:** Death events can happen in each state, where one individual is removed.
- **Swaps between states:** An individual swaps from a state to the next one due to biological evolution or mating.
- **Population control intervention:**

$$u(t) = \sum_i \mathbb{1}_{t_i = t} \mathrm{R}_i.$$

Models without intervention

Simulation with rainfall dependent competition and parameters for *Aedes polynesiensis*[1] without intervention:



Convergence to an ODE system

We define the rescaled process X^K as

$$\mathrm{X}^K(t) = \frac{\mathrm{Z}(t)}{K}.$$

Where $c^K = c/K$, $\mathrm{R}_i = K\boldsymbol{\rho}_i$.
Define the system

$$\begin{cases} \dot{\mathrm{x}}(t) = \sum_{j\in \mathrm{J}} \boldsymbol{\nu}_j a_j(t, \mathrm{x}(t))dt \text{ for } t \neq t_i, \\ \mathrm{x}(t_i) = \mathrm{x}(t_i^-) + \boldsymbol{\rho}_i \text{ for } i \in 1, \dots, r \\ \mathrm{x}(0) = \mathrm{x}_0, \end{cases} \tag{1}$$

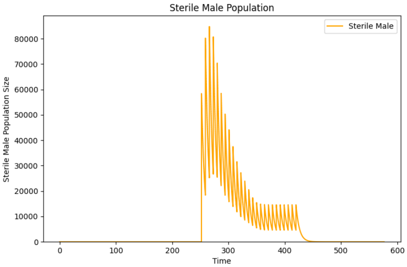
Theorem (Limiting ODE)

Let X^K the rescaled mosquito jump process with $\mathrm{X}_0^K \to x_0$ when $K \to \infty$. Then as $K \to \infty$, the sequence (X^K) converges in probability in $\mathbb{D}([0,T], \mathbb{R}_+^8)$ to the deterministic process x solution of (1).

ODE system

$$
\begin{aligned}
\frac{dE}{dt} &= n_e \beta_E F_f - (\tau_E + \delta_E)E,\\
\frac{dL}{dt} &= \tau_E E - (\tau_L + \delta_L + c(t)L)L,\\
\frac{dP}{dt} &= \tau_L L - (\tau_P + \delta_P)P,\\
\frac{dF}{dt} &= \nu \tau_P P - (1 + \delta_F)F,\\
\frac{dF_f}{dt} &= p(M, M_s)F - \delta_F F_f,\\
\frac{dF_s}{dt} &= (1 - p(M, M_s))F - \delta_F F_s,\\
\frac{dM}{dt} &= (1 - \nu)\tau_P P - \delta_M M,\\
\frac{dM_s}{dt} &= u(t) - \delta_M M_s.
\end{aligned}
$$

Extinction



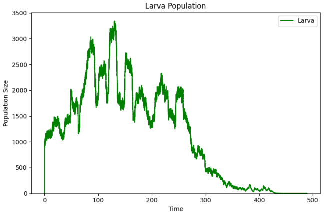


Figure: Larvae Population Stochastic model

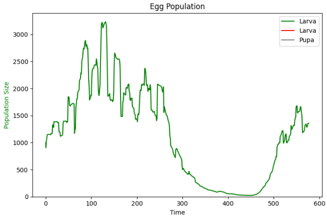
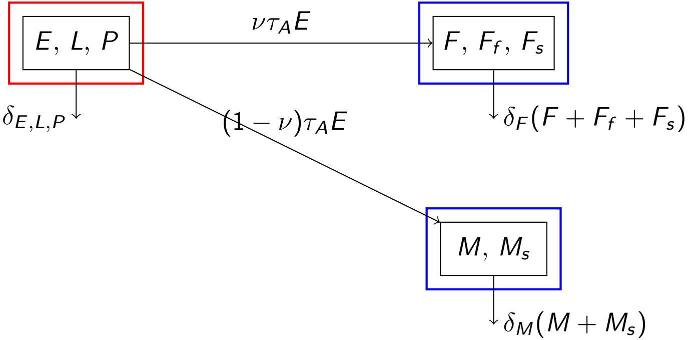


Figure: Larvae Population Deterministic model

Back to Tetiaroa



$E,\ L,\ P$

$\nu\tau_A E$

$F,\ F_f,\ F_s$

$\delta_{E,L,P}$

$(1-\nu)\tau_A E$

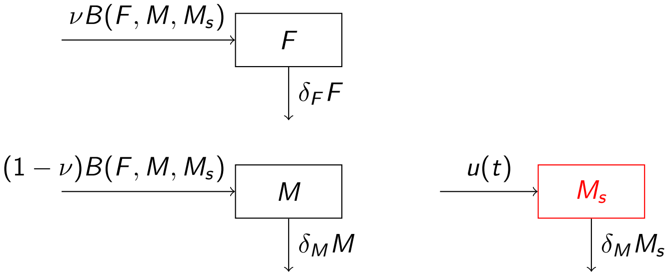
$\delta_F(F+F_f+F_s)$

$M,\ M_s$

$\delta_M(M+M_s)$

In blue: partially observed states (from data), in red: hidden states.

Simpler model



With

$$B(F, M, M_s) = \frac{\sqrt{(\tau_L + \delta_L)^2 + 4c\frac{\beta \xi(M, M_s) p(M, M_s) \tau_E}{\tau_E + \delta_E} F} - (\tau_L + \delta_L)}{2c(\tau_P + \delta_P)} \tau_L$$

Constant environment

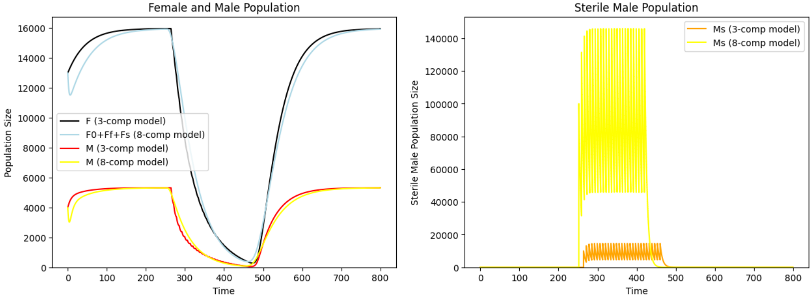


Figure: Eight and three states models for a constant environment with SIT intervention.

Back to Tetiaroa

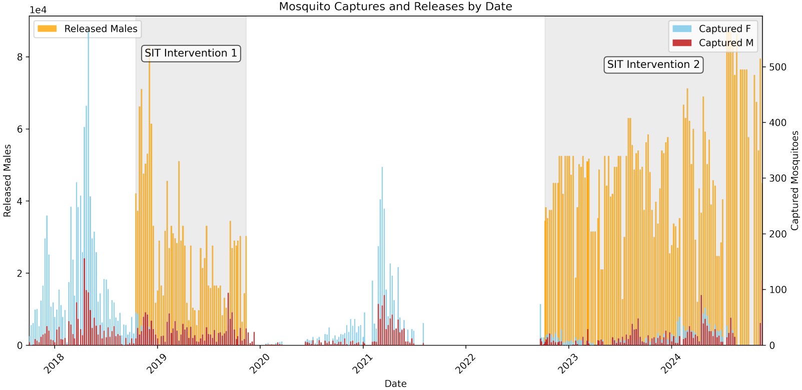


Figure: Captured and released mosquitoes on Onetahi Island.

Two settings

Successful intervention: No more females are captured, we assume the captured individuals are incompatible males released previously. We know the explicit expression of the sterile males:

$$\forall t \in [0, T],\ M_s(t) = \sum_{t_i \leq t} R_i e^{-\delta_M (t - t_i)} \tag{2}$$

Wild population behavior: No introduction of incompatible males, population size is driven by environmental variables.
The three models could be used for estimation of the parameters

Successful intervention

Mecanistic-statistical model The capturable population over a week finishing at t_i is given by

$$C_i = \int_{t_{i-1}}^{t_i} M_s(u)du.$$

The capture at a time t_i is represented by a Poisson realization

$$N_i \sim \mathcal{P}(\alpha C_i),$$

Where α is the capture coefficient.

We want to estimate α and δ_M

MLE

The capturable population formula is explicit

$$C_i = \sum_{j=0}^{i-2} \frac{R_j}{\delta_M}\left(e^{-\delta_M(t_{i-1}-t_j)} - e^{-\delta_M(t_i-t_j)}\right) + \frac{R_{i-1}}{\delta_M}\left(1 - e^{-\delta_M(t_i-t_{i-1})}\right).$$

Probability of k captures:

$$\mathbb{P}_\theta(N_i = k) = \frac{\left(\alpha(C_i - C_{i-1})\right)^k}{k!} e^{-\alpha(C_i - C_{i-1})}.$$

Log likelihood:

$$\mathcal{L}(n_i; \alpha, \delta_M) = \sum_{i=1}^{n} n_i(\log(\alpha) + \log(C_i)) - \alpha \sum C_i.$$

MLE

Test data:

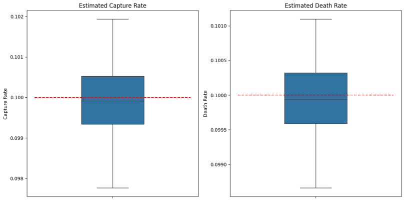


Figure: Simple case: $\alpha = 0.1$ and $\delta_M = 0.1$

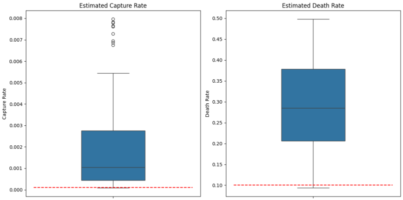


Figure: More realistic case: $\alpha = 0.0001$ and $\delta_M = 0.1$

Field data application:
Form minimize with BFGS :

$$\widehat{\alpha} = 3.599e - 03 \text{ and } \widehat{\delta_M} = 3.193e - 01.$$

Population process

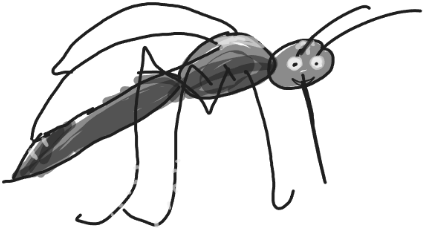
We now need to tackle the full population setting:

$$N_i^M \sim \mathcal{P}\left(\alpha_M C_i^M\right),$$
$$N_i^F \sim \mathcal{P}\left(\alpha_F C_i^F\right).$$

ODE models:
Deterministic approximation (no demographic stochasticity), fast simulation. Many parameters and latent compartments. Mechanistic–statistical framework still applicable?

Stochastic model: Hidden Markov Model. Inference: filtering, EM methods? Implementation: R (pomp), Python?

Thanks for your mosquitention !

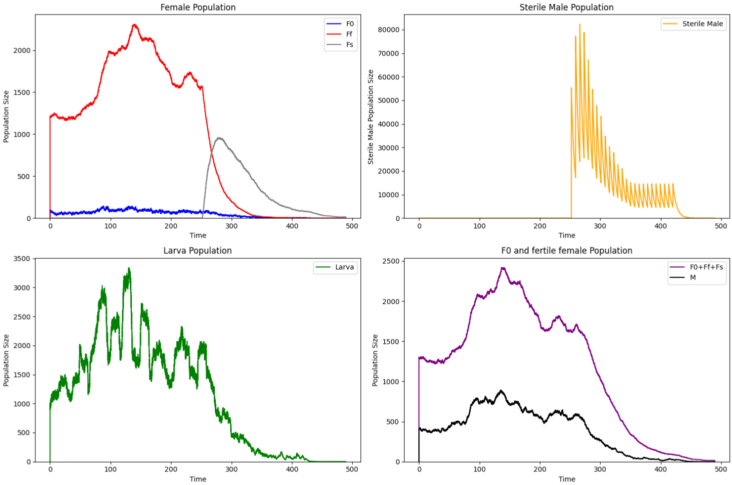


MLE

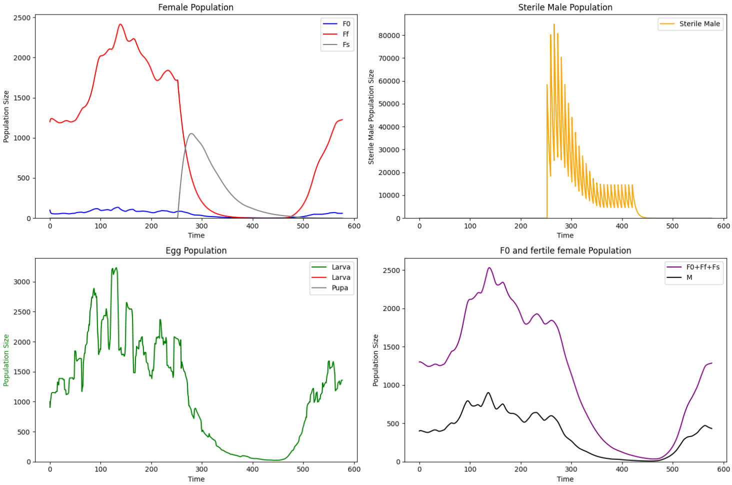
We want an estimation of $\boldsymbol{\alpha}$ and $\boldsymbol{\delta_M}$. Hence, log-likelihood.
Log likelihood:

$$\begin{aligned}
\mathcal{L}(n_i;\alpha,\delta_M) &= \sum_{i=1}^{n} n_i(\log(\alpha)+\log(C_i)) - \alpha \sum C_i \\
&= \sum_{i=1}^{n} n_i \Bigg(\log(\alpha) + \log\left(\sum_{j=0}^{i-2} \frac{R_j}{\delta_M}\left(e^{-\delta_M(t_{i-1}-t_j)} - e^{-\delta_M(t_i-t_j)}\right)\right. \\
&\quad \left. + \frac{R_{i-1}}{\delta_M}\left(1-e^{-\delta_M(t_i-t_{i-1})}\right)\right)\Bigg) \\
&\quad - \alpha \sum \left(\sum_{j=0}^{i-2} \frac{R_j}{\delta_M}\left(e^{-\delta_M(t_{i-1}-t_j)} - e^{-\delta_M(t_i-t_j)}\right)\right. \\
&\quad \left. + \frac{R_{i-1}}{\delta_M}\left(1-e^{-\delta_M(t_i-t_{i-1})}\right)\right)
\end{aligned}$$

Stochastic model with successful intervention



ODE model with failed intervention



SDE Representation

The rescaled population process can be written as

$$\mathrm{X}^K(t) = \mathrm{X}^K(0) + \int_0^t \int_{\mathbb{R}_+} \sum_{j \in \mathrm{J}} \frac{\boldsymbol{\nu}_j}{K} 1_{\{\, r \leq K a_j(s, \mathrm{X}^K(s^-))\,\}}\, Q_j^K(ds, dr) + \sum_{t_i \leq t} \boldsymbol{\rho}_i \tag{3}$$

Where Q_j are independent Poisson random measures.

Convergence to a diffusion

Studying the difference V_K between the birth-death process X^K and the dynamical system x yields a first stochastic diffusion result.

$$V_K(t) = \sqrt{K}(Z^K(t) - X(t))$$

Theorem (Convergence to a stochastic diffusion)

The sequence of processes $(V_K(t))_{t \in [0,T]} \in \mathbb{D}([0, T], \mathbb{R}_+^5)$ *converges as* $K \to \infty$ *to the diffusion process* $(V_t)_{t \in [0,T]}$ *solution of:*

$$V_t = V_0 + \int \partial F(X_s) V_s ds + \sum_j j \int \sqrt{b} dB_s^j$$

Using this theorem, we can approximate our population by the following stochastic diffusion process:

$$Y_t^K = X_t + \frac{1}{\sqrt{K}} V_t,$$

Models with failed intervention