Un Tétris pour générer des précipitations

Rainfall modelling with compound discrete EGPD

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Statistiques au sommet de Rochebrune

Université de Bretagne Occidentale

LMBA

Motivation

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"Fin janvier 2025, la tempête Eowyn, suivie des dépressions Herminia puis Ivo ont apporté des pluies continues. Ces pluies sont tombées sur des sols déjà saturés en eau par un antécédent pluvieux sur l'ensemble du mois, limitant l'infiltration dans le sol, provoquant des débordements de cours d'eau inédits et ralentissant la décrue."

https://www.ille-et-vilaine.gouv.fr

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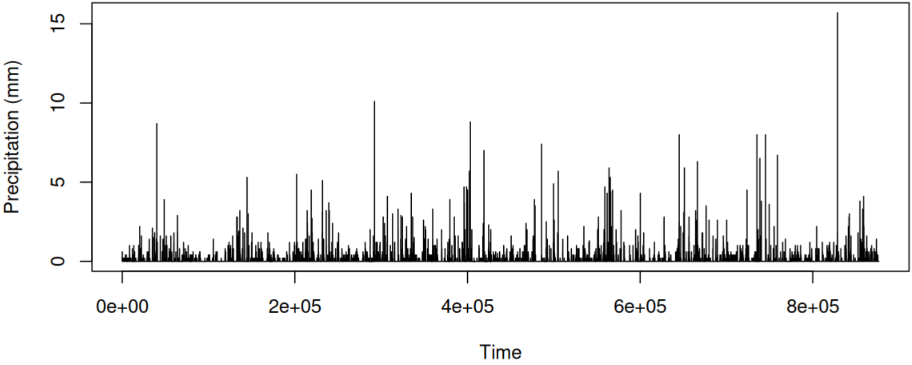
https://www.aspas-nature.org

Goals of this project



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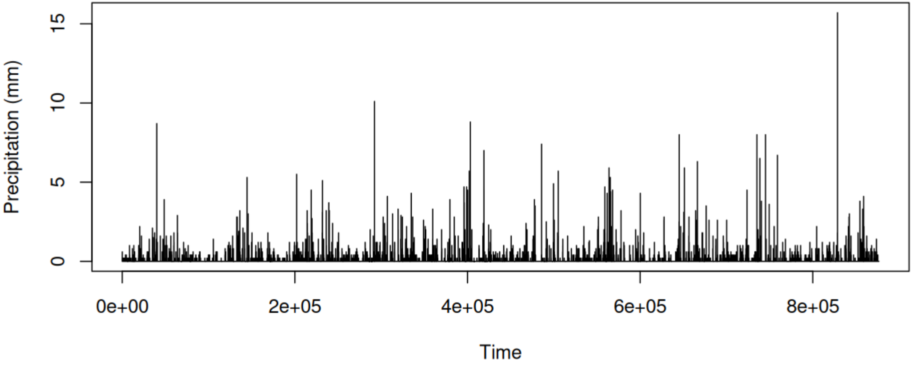

- Model precipitation time series by taking into account time dependence and non-stationarities
- Characterize the whole distribution of rainfall, including non-extreme values
- Develop a stochastic precipitation generator for high-resolution time series (6 minutes)

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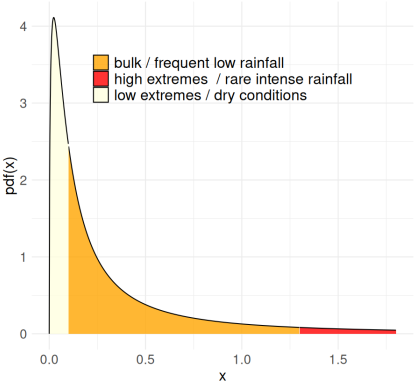
Ex: if $E_i \sim \mathcal{G}amma$, Z is a Poisson-Gamma r.v. (Fisher et al., 1960)

In our case, $E_i \sim$ EGPD (Naveau et al., 2016)

Model for the intensities (or marks)



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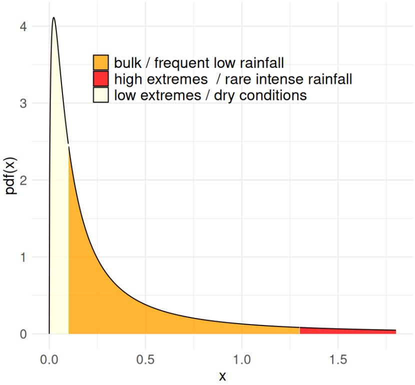

Low extremes
κ the GPD parameter of $1/E$

Bulk
B a CDF function on $[0,1]$

High extremes
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$$E = \sigma H_{\xi}^{-1}\left(\left(B^{-1}(U)\right)^{1/\kappa}\right) \sim EGPD(\sigma, \kappa, \xi, B)$$

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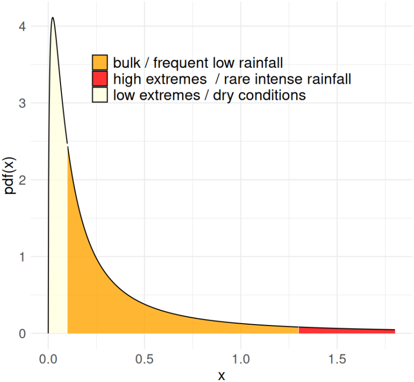

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Sums of *iid* EGPD variables remain EGPD, with same ξ but different B

Adding some temporal dependence: **Trawl processes**[1]

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Trawl $\neq$ Troll !!

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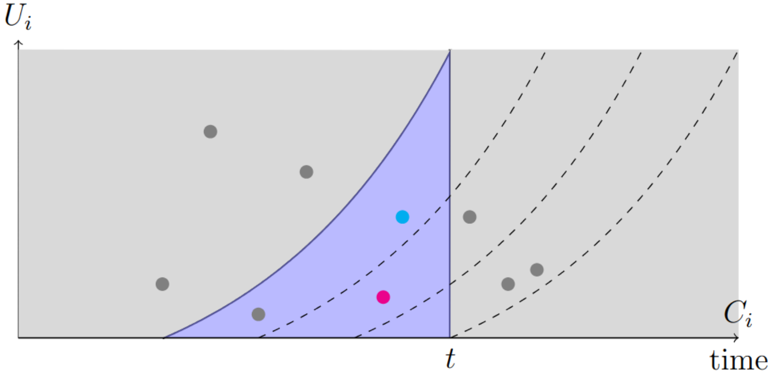

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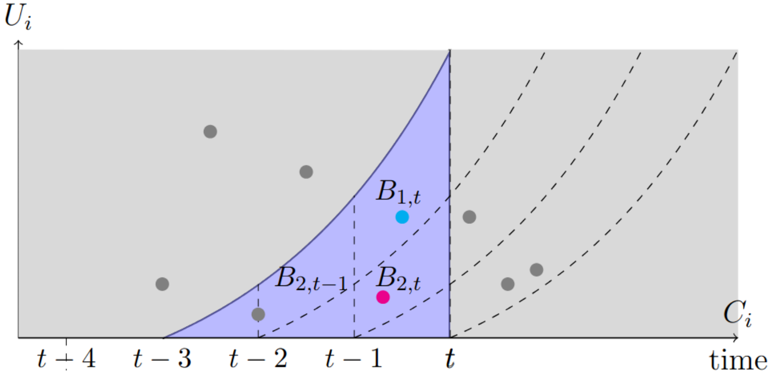

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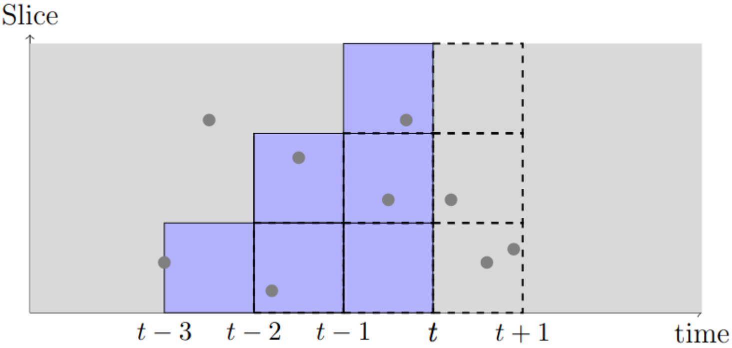

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In each tetris box:

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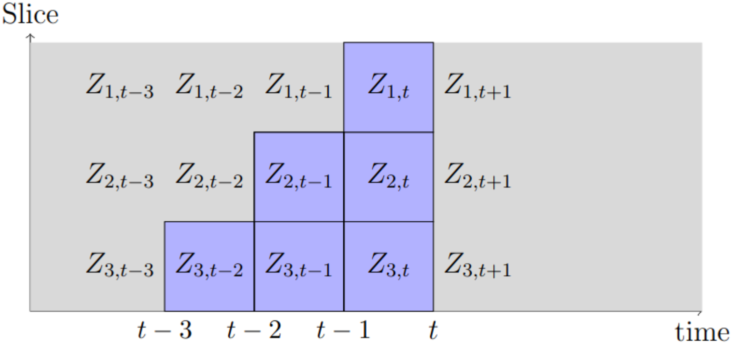

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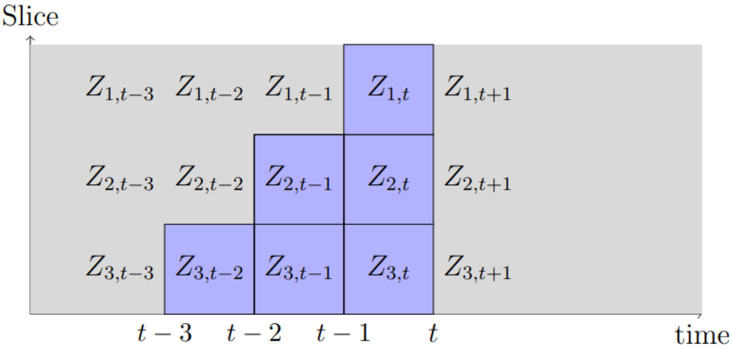

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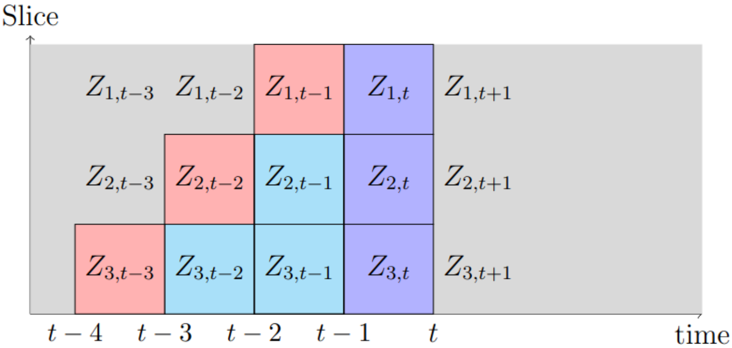

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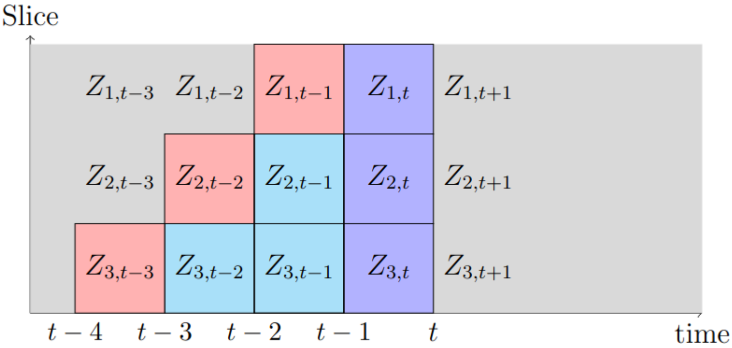

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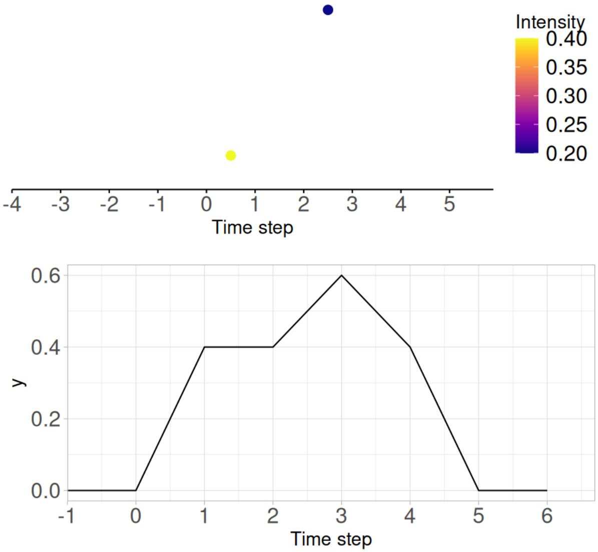

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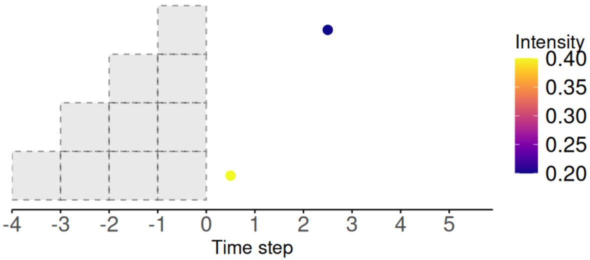

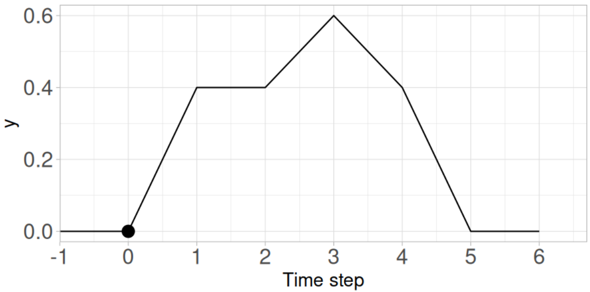

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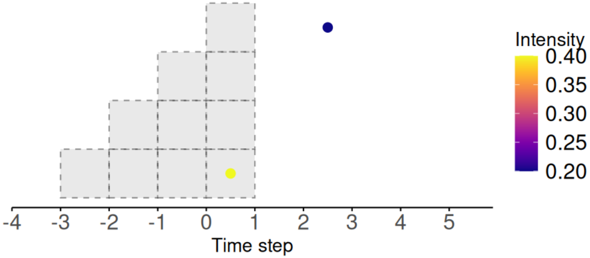

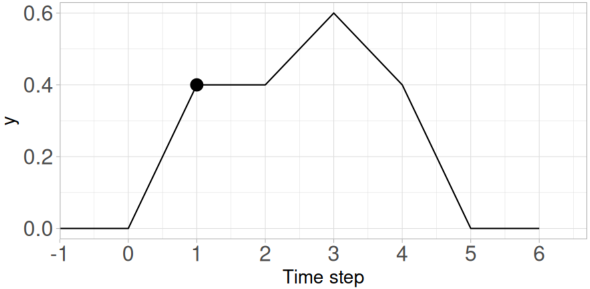

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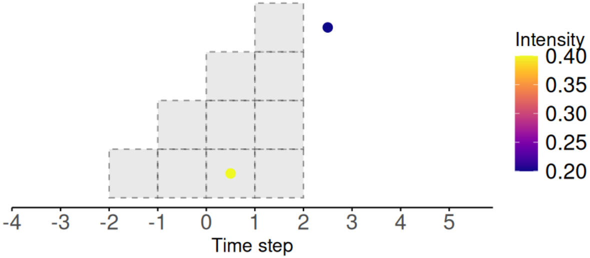

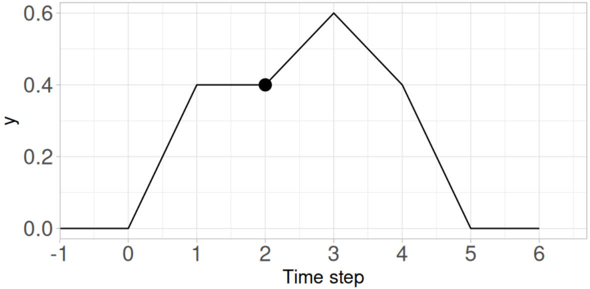

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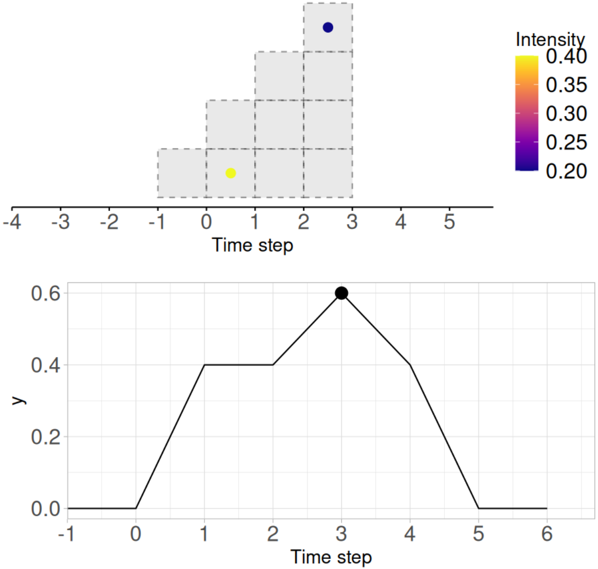

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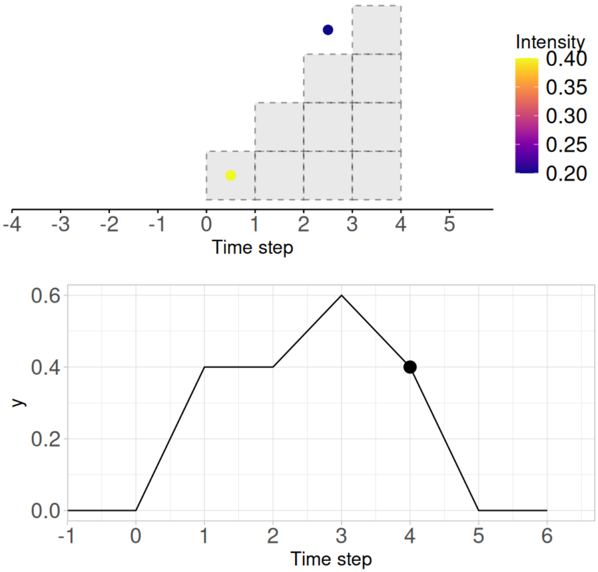

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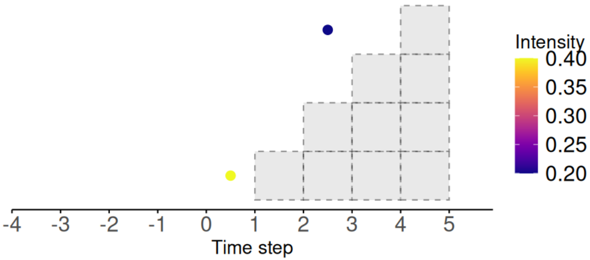

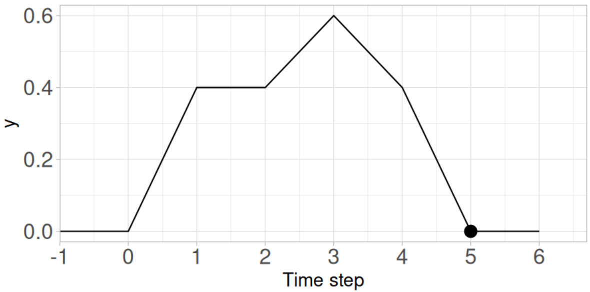

Inference scheme

$$Y_t = \sum_{i \in I_t \setminus I_s} Z_i + \sum_{i \in I_t \cap I_s} Z_i = Y_{(t \setminus s)} + Y_{(t,s)}$$

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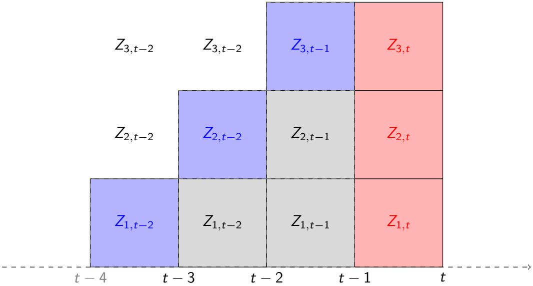

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Bivariate probability function given by

$$\mathbb{P}(Y_t = y_t, Y_s = y_s) = \sum_{k=0}^{\min(y_t, y_s)} \mathbb{P}(Y_{t \setminus s} = y_t - k)\mathbb{P}(Y_{t,s} = k)\mathbb{P}(Y_{s \setminus t} = y_s - k)$$

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- Inference based on pairwise likelihood and Panjer's algorithm [2]

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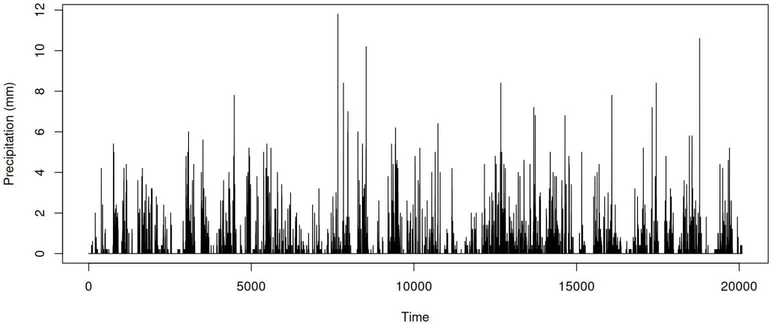
Simulation of "realistic" rainfall events



- Precipitation data from Brest-Guipavas station, France
- 1 hour time resolution
- Covariate: ERA5 precipitation at the nearest grid point
- Effect on the λ_t (Poisson parameters) and on the σ (EGPD scale parameter, by tetris-slice)

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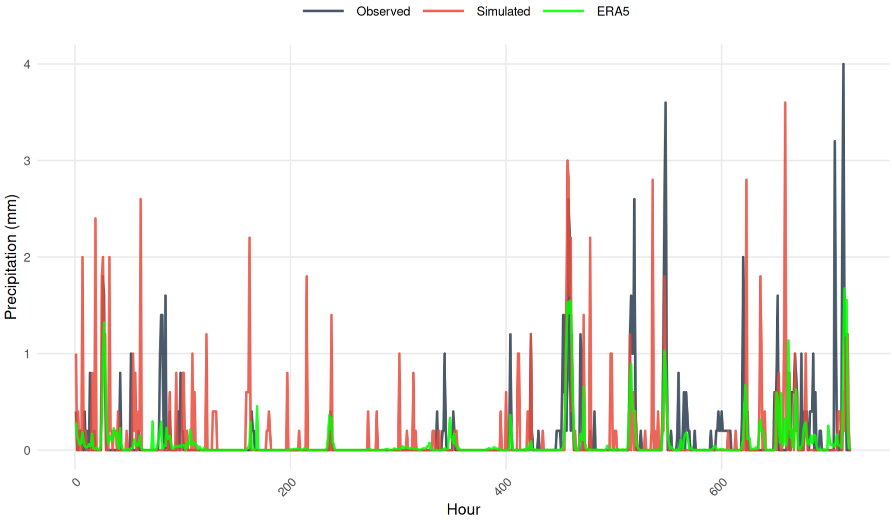

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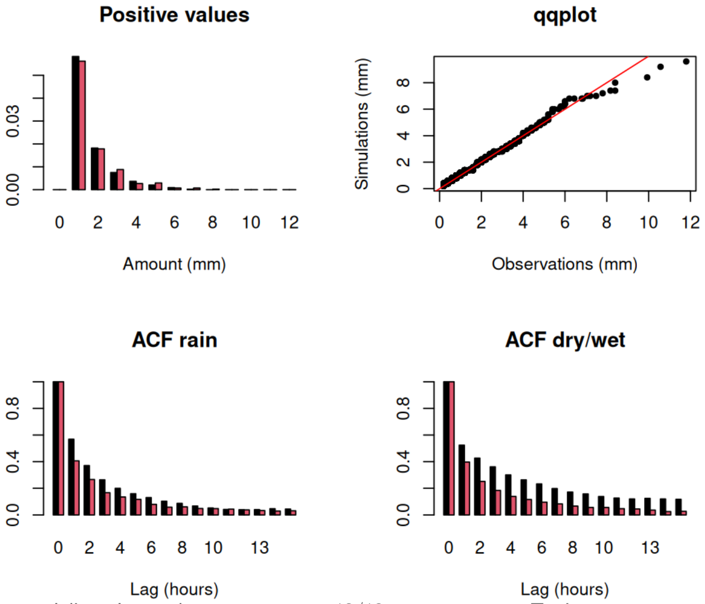

Conclusion and perspectives

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$\rightsquigarrow$ Extension to spatial setting (upcoming PhD)