A first state-switching diffusion model for tiny wasps (trichogramma)

PhD TrichTrack

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Trichogramma : tiny parasitoid wasps

Figure: Female trichogramma on egg of armyworm

- **Egg-parasitoids:** females lay their eggs inside host eggs, which are then consumed from the inside
- **Biological control agents :** released by millions in corn field etc. all over the world

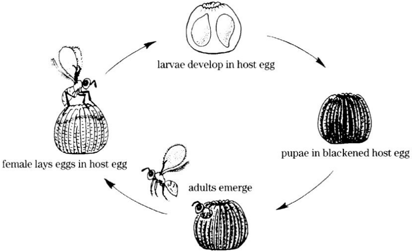

Figure: Trichogramma egg parasitism cycle

- Microscopic wasps (< 0.5mm)

Movement study

- Understanding movement and interaction between individuals will help to understand dispersion in the field
- Their small size makes field observation almost impossible

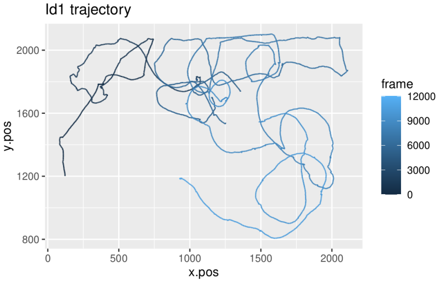

Figure: Example of a trichogramma trajectory

- Insects with very small wings : they are not really able to fly
- Trichogrammas walk and occasionally jump

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Macroscopic study

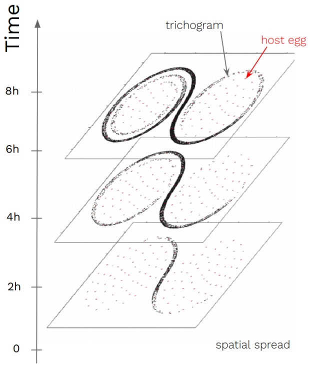


Figure: Double spiral experiment

Experiment from *Burte et al.* (2023) [1] :

- 6m long tunnel (1cm wide and 9mm high) displayed in a double spiral : exceeds trichogramma visual perception range
- 8h experiment, pictures taken every minute
- Thelytokous species : females give birth to genetically identical daughters
- **Main conclusions** :
  - Diffusion seems to be density dependent
  - A behavioural switch occurs over time : individuals alternate between "explorer" and "resident" behaviours.

Microscopic study - video tracking at smaller scale

- Are there truly two distinct movement strategies ?
- How do they move at microscopic scale ?

New experiment : Yuan GAO - PhD Student at INRAE (ISA) and INRIA - 2024-2026

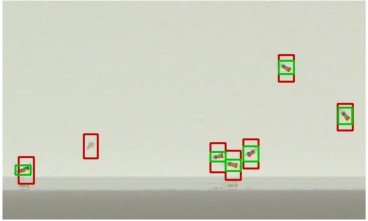
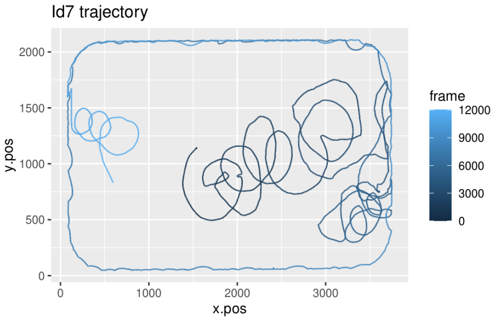


Figure: Video Tracking from Pani et al. 2021 [3]

- 4K 8min videos ; 5cmx9cm arenas
- Thelytokous females, without host
- Individual tracking to reconstruct trajectories
- High and low density videos

Individual trajectories



Illustrative examples of reconstructed trajectories from Yuan Gao's study [2]

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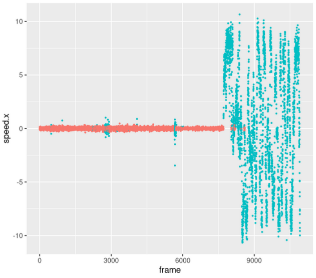
Modelling individual trajectories

- Current analyses rely mostly on empirical observations
- Can we find a mathematical model for the individual trajectories of trichogrammas ?

A first model : State-swicthing model with 2 states on relative speed



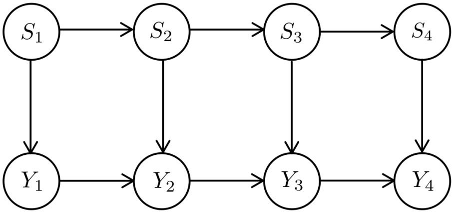


Figure: State-switching model representation

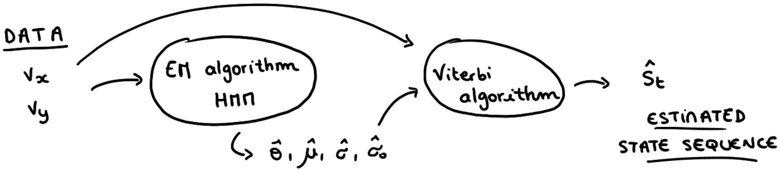
State-switching model on relative speed - model

- S_t 2-value state process / switches only on observation times
- Y_t continuous stochastic observation process (here, $Y_t = V_t^x$ or V_t^y)

Conditional density of our data

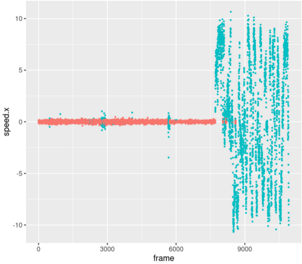
$p(V_t^x, V_t^y | V_{t-1}^x, V_{t-1}^y) = p(V_t^x | V_{t-1}^x) \times p(V_t^y | V_{t-1}^y)$
with

- State 0 : $V_t^x \sim \mathcal{N}(0, \sigma_0^2)$ (resp. Vy)
- State 1 : Ornstein-Uhlenbeck $dV_t^x = \theta(\mu - V_t^x)dt + \sigma dW_t^x$ (resp. Vy)



State-switching model on relative speed - results

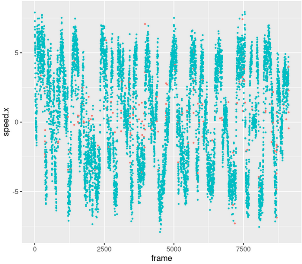
Trich1 : 2-state behaviour



Estimated parameters :

- State 0 : $\hat{\sigma_0} = 0.10$
- State 1 : $\hat{\theta} = 0.02$, $\hat{\mu} = 0.17$, $\hat{\sigma} = 1.24$

Trich4 : 1 state behaviour



Estimated parameters :

- State 0 : $\hat{\sigma_0} = 2.34$
- State 1 : $\hat{\theta} = 0.03$, $\hat{\mu} = 0.09$, $\hat{\sigma} = 0.87$

State-switching model on speed norm - work in progress

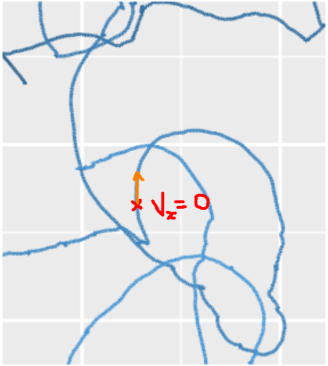


Figure: Example where V_x is equal to 0 but the speed norm is not

Speed norm model

$Y_t = \|v_t\|^2 = V_x^2(t) + V_y^2(t)$, where :

- State 0 : Y_t follows a scaled chi-squared distribution; $Y_t \sim \sigma_0^2 \mathcal{X}_2^2$
- State 1 : Y_t is a CIR process; $dY_t = \kappa(\mu - Y_t)dt + \sigma\sqrt{Y_t}dW_t$

- Explicit likelihood for both states
- The likelihood in State 1 has a complex form : Euler pseudo-likelihood is used for parameter updates

Perspectives

Towards a robust individual-level movement model :

- Finalize the state-switching model based on speed norm
- Explore clustering of individual movement profiles
- Introduce random effects to capture inter-individuality variability

Towards modelling collective movement :

- Assess how local density influences individual behavioural states
- Model dependence between individuals (interaction structure)
- Incorporate jumps into the movement dynamics

Thank you for your attention!

References

[1] Victor Burte et al. "When complex movement yields simple dispersal: behavioural heterogeneity, spatial spread and parasitism in groups of micro-wasps". In: *Movement Ecology* 11.1 (Mar. 2023). DOI: 10.1186/s40462-023-00371-8.

[2] Yuan Gao. *WIP : microscopic trichogramma movement study*. 2025.

[3] Vishal Pani et al. "TrichTrack: Multi-Object Tracking of Small-Scale Trichogramma Wasps". In: *2021 17th IEEE International Conference on Advanced Video and Signal Based Surveillance (AVSS)*. Nov. 2021, pp. 1–8. DOI: 10.1109/AVSS52988.2021.9663814.

Explicit formulas for parameters update - HMM on relative speed

We recall that $p(V_t^x, V_t^y | V_{t-1}^x, V_{t-1}^y) = p(V_t^x | V_{t-1}^x) \times p(V_t^y | V_{t-1}^y)$ with

- State 0 : $V_t^x \sim \mathcal{N}(0, \sigma_0^2)$ (resp. Vy)
- State 1 : Ornstein-Uhlenbeck $dV_t^x = \theta(\mu - V_t^x)dt + \sigma dW_t^x$ (resp. Vy)

We want to maximize complete log-likelihood :

$$Q(\theta;\theta^{[s]}) = \sum_{u=0}^{1} p(V_1 = u | v_{1:T};\theta^{[s]}) \ln \pi_u + \sum_{t=1}^{T}\sum_{u=0}^{1} p(V_t = u | v_{1:T};\theta^{[s]}) \ln \phi_u(v_t | v_{t-1}) + \sum_{t=1}^{T}\sum_{u=0}^{1}\sum_{j=0}^{1} p(V_{t-1} = u, V_t = j | v_{1:T};\theta^{[s]}) \ln A_{u,j} \quad (1)$$

We denote $p(V_t = u | v_{1:T};\theta^{[s]}) = \gamma_t(u;\theta^{[s]})$

Explicit formulas for parameters update - HMM on relative speed

We find that :

$$\hat{\phi} = \frac{\sum_{t=1}^{T} \gamma_t((V_x(t) - \bar{V})(V_x(t-1) - \tilde{V}) + (V_y(t) - \bar{V})(V_y(t-1) - \tilde{V}))}{\sum_{t=1}^{T} \gamma_t((V_x(t-1) - \tilde{V})^2 + (V_y(t-1) - \tilde{V})^2)}$$

$$\hat{\mu} = \frac{1}{2(1-\hat{\phi})} \frac{\sum_{t=1}^{T} \gamma_t(V_x(t) + V_y(t) - (V_x(t-1) + V_y(t-1))\hat{\phi})}{\sum_{t=1}^{T} \gamma_t}$$

$$\hat{\sigma}_1^2 = \frac{\hat{\theta}}{1-e^{-2\hat{\theta}}} \frac{\sum_{t=1}^{T} \gamma_t((V_x(t)-\hat{\mu}-(V_x(t-1)-\hat{\mu})e^{-\hat{\theta}})^2+(V_y(t)-\hat{\mu}-(V_y(t-1)-\hat{\mu})e^{-\hat{\theta}})^2)}{\sum_{t=1}^{T} \gamma_t}$$

$$\hat{\sigma}_0^2 = \frac{\sum_{t=1}^{T} \gamma_t(0;\theta^{[s]})(V_x(t)^2 + V_y(t)^2)}{2\sum_{t=1}^{T} \gamma_t(0;\theta^{[s]})}$$

Extended results - Relative speed